DISCUSSION BEFORE THE RADIO SECTION, 16TH NOVEMBER, 1949

Mr. D. A. Ley: The use of the author's stability margin is a most excellent suggestion. There is usually considerable difficulty in deciding just how stable an amplifier is when produced in large quantities, and sometimes the practice of decreasing the feedback by 6 db to see whether the amplifier is still stable can be adopted. This is not always possible, however, for in some types of amplifiers the feedback circuit cannot be altered very conveniently. The author's suggestion of plotting feedback and calculating the stability margin has been found to be a useful method which gives sound results.

Mr. H. D. Ellis: A straightforward amplifier with a simple feedback loop developed in accordance with Dr. Bode's precepts represents to me good sound practice. The author has extended this method in an admirable fashion, but I feel a little uneasy about it and submit that the author's technique is perhaps applicable only to special classes of amplifiers. I do not like the idea of an amplifier becoming unstable under certain conditions which are expected to occur in practice. I feel, too, that the maintenance engineer and the production testing engineer may well be in difficulties if they meet one of the amplifiers such as the author has shown us which has gone wrong.

I therefore put forward a plea for feedback amplifiers to be designed on a simple basis, and suggest that the author's extended application should be used only where it becomes absolutely necessary. That it is necessary in some cases I have no doubt; I can think of one in particular where feedback is required from the end of a modulated radio transmission channel back to the input, and certain phase changes occur which cannot be avoided.

A valuable part of the paper is that concerned with double loops. In an amplifier which is designed to go down to low frequencies, there are apt to be important feedback paths through the decoupling components, and it seems to me that this method of treatment is excellent in determining the stability margins in these cases.

I should like to mention some of the difficulties which I have experienced in using Dr. Bode's technique in measuring the response of amplifiers at low frequencies. It is all very well to demand an amplifier that shall go down to perhaps 20 c/s, with 40 db of feedback, but the response then has to be measured some 5 octaves below 20 c/s, i.e. at less than 1 c/s. It is not easy to get an oscillator in the first place which gives a sinusoidal waveform at those frequencies, or to get a measuring instrument which will respond to them. I should value the findings of other speakers who may perhaps have met this problem. As far as I know, the cathode-ray tube provides the only method of measurement.

Another matter in the testing of feedback amplifiers which has caused some trouble is in determining the correct matching impedances when the feedback loop is broken into in order to do measurements; the ability to match precisely the output circuit by the impedance which it normally encounters at the point where the break has been made, and the provision of a zero-impedance sending source to feed the input, are not always easily achieved.

Mr. F. J. M. Laver: The design of feedback amplifiers is often described as an art, and it certainly offers scope for much individual skill, both in specification and in design.

Consider first the specification: a major use of feedback amplifiers is as cable repeating amplifiers; but the advantages which they confer can sometimes be obtained in other ways. Thus, to take only the most obvious example, stability of gain is important in a cable circuit that may have 20 or more amplifiers in series, and may be obtained by feedback. Alternatively, gain stability may be achieved by the use of stabilized power supplies, or the circuit gain may be controlled automatically or a system of transmission may be used that is not sensitive to gain changes. The problem of deciding between such alternatives is primarily an economic one; moreover, they rarely present themselves as simple alternatives, and the scope for art in the specification of system components is wide indeed. Again, in the detailed design of an amplifier a given performance may be variously obtained and there is much to test the designer's skill. The author has made a substantial contribution to the difficult art of the designer by conferring an increased flexibility in the manipulation of the characteristic of an amplifier outside its pass band.

The author describes a method for controlling the oscillation of a conditionally stable amplifier by arranging that its stages overload in a chosen sequence. Does the sequence in which overload occurs alter as the valves age, leading to unquenched oscillations?

When determining the overload point of feedback amplifiers, we have noticed that with an input signal of critical amplitude the amplifier may oscillate: we think it due to a second harmonic produced in an overloaded stage, which intermodulates with the signal in a later stage to produce a fundamental component having a phase correct for oscillation. It is easy to miss this oscillation if the amplifier is tested by increasing the input signal in steps, for example of 2 db, and I should like to know whether the author has observed any similar phenomenon.

With high-frequency feedback amplifiers we have found the idea of stability margin useful because the $\mu\beta$ characteristic cannot be accurately measured. In this connection I should like the author's comments on the possibility of extending his rather complex technique to frequencies of several megacycles per second, where stray reactances are difficult to measure and control.

Mr. D. A. Bell: I was glad to read the paper because I had also become dissatisfied with the phase margin as a stability criterion, although I had not produced a substitute for it.

As the practical applications in which I am interested usually involve more than two stages, one cannot say that the limiting phase shift is 180°. Therefore, if one accepts what I take to be the author's terminology, the amplifier must be conditionally stable; but in my view "conditionally stable" means that part of the characteristic passes outside the critical point, whereas one can alternatively design so that the characteristic passes inside the critical point. In other words, the normal gain is less than unity at 180°, and I regard this as being unconditionally stable.

It seems to be assumed that Dr. Bode dealt only with phase margin, but my impression is that he points out that if 180° must be crossed, amplitude margin as well as phase margin is considered, and it is a matter of experience as to which of the two shall be used and what magnitude of margin is necessary. My particular interest in this arose from the application of precisely similar concepts and mathematics to another type of

loop known as a servo system, in which the response requirement is rather different.

For a typical two-stage amplifier the plot is asymptotic to the 180° axis, and there is some phase margin. If there were more than two stages, the plot would cross the axis and one could speak of amplitude margin. The characteristic in Fig. A repre-

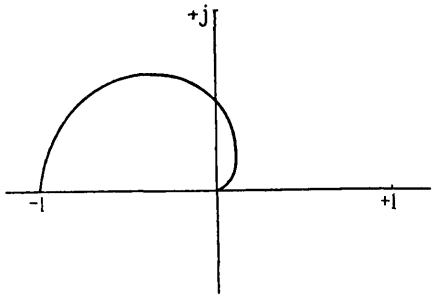

Fig. A.—A Nyquist plot of a critically damped resonator.

sents a Nyquist plot of a critically damped resonator. How do we know that that represents critical damping, while neighbouring loci on either side represent over-damping and under-damping? Perhaps the author's criterion of stability margin ought to give us the answer; but I doubt this, because I am sure the transient response corresponding to Fig. 9(III) will not be critically damped when it has the same phase margin as Fig. A. Therefore, our "damping margin" has to be some sort of integrated distance from the critical point to a considerable region of the curve, rather than the distance to the nearest point, which is the stability margin.

It seems to me, therefore, that there are two possible design criteria: one is in terms of frequency and gain, for a flat working band; and the other is in terms of transient response for a sharp rise and not much overshoot. The author's method of design covers only the former. It also seems that the author's method requires that the first stage, μ_1 in Fig. 8(*a*), should have useful gain outside the working band, so that $\mu_1\beta_1$ can operate outside the band.

Cannot the question of compound feedback be simplified? If all feeds can be broken at one point, it should be possible to regard the system as made up of units each modified by local feedback. In the top part of Fig. 8 it seems to be possible to substitute the constant μ_1' for μ_1 with feedback, to put that in tandem with μ_2, and then to apply feedback overall. Applying this at the end of Section 3, where the $\mu\beta$'s are large and correspond to the characteristic depending on the feedback path only, the reduction in gain depends on $\mu_1\beta_1$ plus $\mu_2\beta_3$, and the reduction in distortion depends only on $\mu_1'\mu_2\beta_3$. This might be expected, since only the $\mu_2\beta_3$ loop encloses the source of distortion.

Dr. K. R. Sturley: About 18 months ago I had to analyse for teaching purposes a circuit consisting of combined current and voltage feedback, the results of which have been published elsewhere.* But I missed the important point brought out by the author, namely that subsidiary feedback can improve stability outside the pass region. Re-examination of the original analysis has shown an easy way of proving this, as follows. Suppose an audio-frequency amplifier contains two stages of gain, A_1 and A_2, over both of which voltage negative-feedback operates. Then, overall amplification $= A_1A_2/(1 + A_1A_2\beta)$, where β = feedback factor.

Over the pass range, amplification $= 1/\beta$ because $A_1A_2\beta \gg 1$.

Outside the pass range instability occurs when $A_1A_2\beta = -1$.

* "Combined Current and Voltage Feedback," *Electronic Engineering*, May 1949, p. 159.

If current feedback is applied to the A_1 stage across a cathode resistor R, overall amplification is given by

$$\frac{\frac{A_1}{1 + g_mR}A_2}{1 + \frac{A_1}{1 + g_mR}A_2\beta} = \frac{A_1A_2}{1 + g_mR + A_1A_2\beta}$$

Over the pass range $A_1A_2\beta/(1 + g_mR) \gg 1$, so that amplification is still $1/\beta$. Outside the pass range instability occurs when $A_1A_2\beta = -(1 + g_mR)$.

Hence the possibility of instability is much less in the circuit with subsidiary feedback.

Mr. P. P. Eckersley: Many years ago, engineers were struggling to apply positive feedback to make circuits oscillate; nowadays it seems equally difficult to apply negative feedback to stop them oscillating. The light that Dr. Bode provided has illuminated the path we should follow, but experience proves that it is far from easy to keep to it.

I should like to have the author's guidance as to whether it is better to measure attenuation round the feedback loop and then calculate phase change, or to side-step the labour of calculation and measure phase. The answer to this question when put to various experts has been about equal on one side and the other; some engineers are happier to take a reading than to accept the results of calculation; vice versa, others say, Why saddle yourself with the difficulties of phase measurement when an attenuation graph and some calculation work are all that you need? It is pertinent to observe that the author has provided a graphical method to ease the burden of calculation, and it seems by implication that he would be prone to dispense with phase measurement. If anyone were to come down firmly on the side of calculation, could his arguments be set aside by the knowledge that such calculations apply only for minimum-phase-shift networks, and that one cannot be sure that these are necessarily in the circuit? Therefore is it not safer to measure?

At the risk of demonstrating a failure to understand the paper in its application to subsidiary loops, I should like to ask why the whole problem may not be based on the Bode step theory, in which the overall feedback-loop attenuation characteristic has to conform to a certain shape with the steps at the extremes of frequency. Thus if any circuit is introduced in any stage of the amplifier proper so that the gain exhibits a change at any band of frequencies outside the working range, to produce the necessary step in the characteristic, can this not be read on to the Bode criteria without introducing the concept of a "subsidiary loop"? It may be that the author feels there are better criteria than achieving the Bode step characteristic, in which case, of course, my question does not arise.

Mr. I. F. Macdiarmid: I should like to make a few remarks concerning the idealized $\mu\beta$ locus shown as curve OXABCD in Fig. 7. This form of locus is suggested as a basis for design, but, in single-loop amplifiers at any rate, it may lead to a serious danger of instability.

The author proposes that the $\mu\beta$ characteristic should be designed with valves at the upper limit of their gain tolerances. The normal tolerance on valve mutual conductance is $\pm$ 2 or $\pm$ 3 db, so that a 3-stage amplifier with normal valves will have a gain of some 6–9 db less than that for which the $\mu\beta$ locus has been calculated. The actual $\mu\beta$ locus will be as shown in Fig. B, where it can be seen that the "constant stability margin" has been lost and the working phase margin is only 15°. As pointed out in the paper, this margin is sufficient to prevent instability, provided the component tolerances do not exceed $\pm$ 10%.

The components which have most effect on the characteristic in the critical region are valve inter-electrode capacitances and

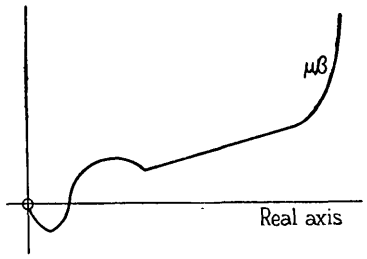

Fig. B.—The $\mu\beta$ locus of Fig. 7, but with normal valves.

stray capacitances. The normal specified tolerance on valve capacitances is ± 15%, and it seems unlikely that circuit strays can be held to closer limits. The probable variation in capacitance is thus about ± 15% and not the ± 10% required by the author. Using the method of Part 2, Section 3, this capacitance variation can be shown to produce a phase variation of about 4° per *RC* coupling, which amounts to 12° in a 3-stage circuit. Consequently, with normal valves and adverse capacitances, the $\mu\beta$ locus can pass within 3° of the point (1, 0), which, I think the author will agree, is rather close.

By modifying the author's design procedure slightly it is possible to overcome this difficulty. The modification is to include a separate gain margin in the manner suggested by Bode. To do this the $\mu\beta$ characteristic is designed to be similar to that shown as curve OXABCD in Fig. 7, but using valves of normal gain rather than high gain. Instead of allowing the small arc to cut the real axis immediately, i.e. at (0·5, 0), a gain margin must be included. The actual figure will depend on the valves being used, but for a 3-stage amplifier it will normally be of the order of 9 or 10 db, in which case the $\mu\beta$ locus must not cut the real axis until $|\mu\beta| < 0{\cdot}3$. The type of characteristic so obtained is illustrated in Fig. C. This characteristic does not give what

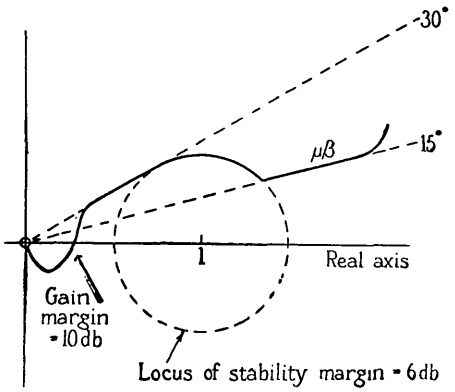

Fig. C.—The modified $\mu\beta$ locus with normal valves and 10-db gain margin.

the author calls a "constant stability margin," but I think it will give a greater margin against instability under average conditions than the characteristic he suggests.

Mr. P. G. Redgment: I should like to ask the author whether he has any experience of these conditionally stable amplifiers in service under ordinary industrial conditions; and, if so, whether he can give any information as to general serviceability, including the incidence of, and the time taken to clear, faults, as compared with the simpler types in general use.

Mr. D. H. Hughes: I was disappointed to find that the paper is concerned only with amplifiers of the minimum-phase-shift type. There are some important instances where excess phase shift is unavoidable: to quote two examples, there are frequency-modulation receivers employing modulation feedback, and radio-frequency transmitters with envelope feedback. Dr. Bode deals only briefly with these and indicates that it is necessary to add a number of extra valve stages in order to achieve stability. Has the author had any experience of such cases, and can he suggest any more economical method of dealing with them?

Mr. I. J. P. James: In Fig. 2 the author refers to Wheeler networks, and it would appear that one has to go to the United States to find out about a circuit which was developed in this country. An identical circuit was disclosed by W. S. Percival in British Patent 475490, dated February, 1936, which deals with the placing of extra capacitance across a filter to maintain its input impedance constant.

With regard to Fig. 27, I should like to ask the author at what signal level the characteristic was taken. It would be interesting to know the shape of the curve at different input levels to indicate whether the change in $\mu\beta$ with level affected the frequency response.

Mr. J. H. Westcott: The question of dealing with excessive phase shift has been raised. The solution to this problem is unlikely to be arrived at by intuition; it is necessary to create an unstable subsidiary loop, a seemingly drastic measure. The justification for such a measure is provided by a mapping theorem in the theory of functions of a complex variable, of which Nyquist's theorem is a special case. The method essentially is to employ the excessive phase in creating the unstable subsidiary loop, but to connect this loop into the system in such a manner that the overall system is stable. The design work, while somewhat involved, can result in a system of exceptional quality.

Mr. F. E. Rogers: One or two speakers have made observations on the desirability of measuring accurately the gain and phase shift of an amplifier.

I should like to indicate a simple method of measurement which I have applied to the cathode-follower amplifier. I think that the principle involved, which is based on balancing the output voltage against a fraction of the input signal, might also be applied to an amplifier having a complex gain whose modulus is greater than unity.

The measurement of cathode-follower gain is illustrated in Fig. D. The theory of the circuit shows that, in the absence of

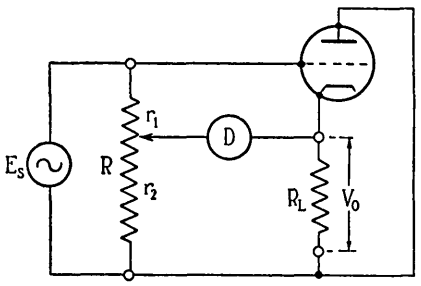

Fig. D.—Basic circuit for measurement of cathode-follower voltage gain.

stray reactances, the potential across the cathode load resistor R_L is in phase with the input signal E_s. Hence, by adjustment of the voltage divider R, a null in the detector current may be obtained. The voltage gain of the amplifier is then simply

$$\left|\frac{V_0}{E_s}\right| = \frac{r_2}{r_1 + r_2}$$

If there is appreciable phase shift, the circuit may be modified by the addition of a condenser across either r_1 or r_2, depending upon the sign of the angle.

The application of the method to a normal amplifier is illus-

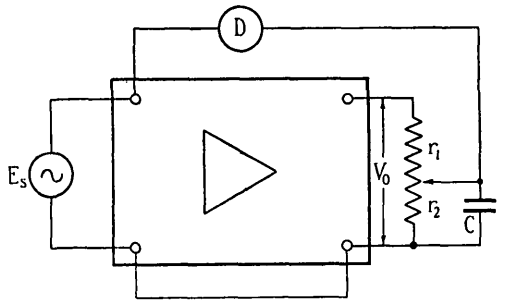

Fig. E.—Basic circuit for measurement of voltage gain and phase shift of a normal amplifier.

trated in Fig. E. In this case, a fraction of the output voltage is balanced against the input. In the illustration it is supposed that there is an even number of valves in cascade, so that the output voltage V_0 is nominally in phase with the input signal E_s. The phase shift due to stray reactances and inter-valve coupling networks is corrected by the condenser C. When balance is achieved, the gain is given by

$$\left|\frac{V_0}{E_s}\right| = \sqrt{\left[\left(\frac{r_1}{r_2} + 1\right)^2 + (\omega C r_1)^2\right]}$$

and the phase shift by

$$\theta = \text{arc tan } \frac{\omega C r_1 r_2}{r_1 + r_2}$$

In a test at 50 c/s on a 2-stage resistance-capacitance-coupled amplifier, using a vibration galvanometer as detector, agreement of the order of 1 % was obtained between measured and calculated values.

There is a risk that oscillation may occur at some frequency other than the test value, owing to the feedback path provided by the detector. Although the amplifier tested appeared quite stable, it is suggested that the inclusion of a tunable filter in series with the detector would ensure stability.

Mr. M. G. Scroggie (*communicated*): With the limited number of symbols available it is no doubt unavoidable that some of them are given more than one meaning. But it is surely an understood principle that such meanings should be sufficiently distinct from one another to create no risk of confusion. Thus, in contexts where μ is used to mean "permeability" it is very unlikely that it would be taken to mean "micro-" or "amplification factor." In a paper on amplifiers, however, one might reasonably expect it to stand for "valve amplification factor," that being a British and American standard, following the practice of at least three decades. The non-standard use of μ to denote gain seems both undesirable and unnecessary, and has not even the excuse of any particular aptness.

The symbol A (or α) has been generally used to denote amplification or gain, especially in negative-feedback contexts, and is more apt than the already hard-worked μ. It would be interesting to know the author's reasons for preferring μ, and how he distinguishes amplification factor from gain in expressions containing both.

* DUERDOTH, W. T.: "A Phase Meter for the Frequency Band 100 kc/s–20 Mc/s," *Post Office Electrical Engineers' Journal*, 1949, 42, p. 43.

* To be published in the *Wireless Engineer*.

† ESPLEY, D. C., CHERRY, E. C., and LEVY, M. M.: "The Pulse-Testing of Wide-Band Networks," *Journal I.E.E.*, 1946, 93, Part IIIA, p. 1176.